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Abstract

We investigate extensions of the classic Rothschild and Stiglitz (1976) (RS) model of adverse selection under asymmetric information. In RS, low-risk customers are worse off owing to an externality created by high-risk buyers in the market. We find critical changes in insurance buyers' behavior under the joint assumptions of transaction costs and buyer heterogeneity with respect to either risk aversion or wealth. Combining transaction costs and heterogeneity, we find a separating equilibrium in which neither high-risk nor low-risk individuals are penalized due to information asymmetry.

Introduction

The impact of asymmetric information on markets is an important issue in the economics literature. In 2001, Joseph Stiglitz, George Akerlof, and Michael Spence were awarded the Nobel Prize in Economics for their analyses of markets under asymmetric information. In their seminal paper, Rothschild and Stiglitz (1976) (RS) provide a framework for analyzing the problem of adverse selection in insurance markets. Their central finding is that information asymmetry causes markets to deteriorate. When a separating equilibrium exists, "The high-risk (low ability, etc.) individuals [exert] a dissipative externality on the low-risk (high ability) individuals,"[1] so that the low-risk customers are worse off due to the presence of high-risk customers. Furthermore, RS find that competitive insurance markets may have no equilibrium.

In spite of the pessimistic conclusions of RS, we observe that many markets function well under asymmetric information: labor markets, credit markets, and security

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[1] Rothschild and Stiglitz (1976, p. 468).

markets are obvious examples. Moreover, the results of empirical studies are not always consistent with the RS predictions. For example, in RS, high-risk insurance customers always buy more insurance than low-risk customers. However, Chiappori and Salanié (2000) find no statistical evidence that customers with higher accident probabilities purchase contracts with more comprehensive coverage. In this article, we modify several of RS's assumptions and offer new insights into why the RS results are not always consistent with empirical findings.

The most important contribution of this article is that we show conditions under which markets with asymmetric information remain efficient. Whereas RS assume that there are no transaction costs in the insurance markets and that customers are identical except for loss probability, we evaluate equilibrium when there are transaction costs and when customers differ with respect to risk aversion or endowment, in addition to loss probability. In the RS framework, all customers prefer full coverage. In our setting, when there are proportional costs, all buyers prefer partial coverage. Our assumptions of heterogeneity with respect to risk aversion or wealth indicate that relevant factors other than loss probability play a part in the insurance decision. We find that if customers are different enough in these new dimensions, and if there are transaction costs, a separating equilibrium exists in which there are no negative externalities.

This article contributes to two areas of the literature that have evolved from RS. First, it extends previous work on the effect of transaction costs in insurance markets. Many papers investigate optimal choice when the premium contains a fixed-percent premium loading. Arrow (1971) uses calculus of variations to show the optimality of a deductible, whereas Mossin (1968) proposes a theorem stating that partial insurance is optimal. In another important study, Dionne, Gourieroux, and Vanasse (1998) make an attempt to show that the RS result remains unchanged when different risk types have the same proportional cost structure. We integrate transaction costs into the RS framework by systematically evaluating two cost structures: constant costs and proportional costs. We argue that high-risk individuals suffer more from proportional costs than low-risk individuals, and the RS externality result persists.

Second, our study extends the literature on heterogeneity among insurance customers. The RS model assumes that people are homogeneous in risk aversion, endowment wealth, and loss severity. These assumptions may not be realistic, and we offer an alternative setting that yields new insights.[2]

Heterogeneous risk aversion has been previously evaluated by De Meza and Webb (2001). They assume that risk-averse people tend to buy more insurance coverage, whereas reckless people put less value on insurance protection. In their setting, it is possible to reach a partial pooling equilibrium, in which some, but not all, low-risk individuals are worse off. In our model, heterogeneous risk aversion has a different

[2] Our article is closely related to Doherty and Jung (1993). They modify the RS model by considering severity differences in addition to probability and their conclusion is that first-best solutions are feasible. Although Doherty and Jung concentrate on a single assumption, we focus on the externality conclusion of the RS model by revising multiple assumptions.

effect: we find that the market may reach a new separating equilibrium and the information problem becomes irrelevant. This result, however, relies on the simultaneous presence of proportional costs and heterogeneous risk aversion.

In our framework with proportional costs, we analyze two scenarios: Scenario A, in which high-risk customers are more risk averse (less wealthy) than low-risk customers, and Scenario B, in which low-risk customers are more risk averse (wealthier) than high-risk customers. In Scenario A, when the difference in risk aversion (wealth) between high- and low-risk customers is great enough, the optimal contracts of the high- and low-risk customers will diverge, creating a separating equilibrium. In the new separating equilibrium, neither risk group imposes a negative externality on the other group. Henceforth, we call this an NE (no externality) equilibrium. In Scenario B, an NE equilibrium exists when the difference in risk aversion (wealth) is large and the difference in loss probabilities is small. Moreover, if consumers differ in wealth, rather than in risk aversion, the same no-externality result occurs (high wealth is equivalent to low-risk aversion).

It appears that the RS negative externality conclusion depends on the assumption that high- and low-risk customers are identical on all dimensions other than loss probability; customers are so similar that the risk groups prefer the same policy. They do not have other reasons for preferring policies that are different. Our results suggest that the interaction between transaction costs and consumer heterogeneity may explain, at least in part, why we observe markets thriving under asymmetric information and why empirical results do not always confirm the RS predictions.

The remainder of this article is organized as follows: first is an overview of the RS model, followed by an analysis of the effect of transaction costs in the next section. The case of heterogeneous risk aversion and the NE equilibrium are presented next, and then the effect of heterogeneous wealth is investigated. Finally, conclusions are presented.

THE MODEL

Because our analysis is based on the RS model, we begin with a brief review of their model and its assumptions. There are two types of insurance customers in the market, low-risk and high-risk. They have different accident probabilities, $p^{\mathrm{H}} > p^{\mathrm{L}}$. Customers know their accident probabilities, but the insurance company does not. RS also make the following four simplifying assumptions:

1. There are no transaction costs and the insurer makes zero expected profit.
2. Both types of customers have the same level of risk aversion.
3. Every client has an identical endowment W.
4. If there is an accident, both risk groups have the identical loss amount d.

In this article, we modify the first three assumptions, thus making the two groups more dissimilar than they are in the original model. We adopt the following assumptions from the RS model.

- The insurance company writes a contract $\alpha = (\alpha_1, \alpha_2)$, where α_1 is the premium amount and α_2 the net indemnity amount (premium is deducted). If an accident occurs, the individual will receive a total indemnity of $\alpha_1 + \alpha_2$. The price of insurance is $q(\alpha) = \alpha_1/\alpha_2$. Since neither the insurance premium nor the indemnity should be negative, we call an insurance contract (α_1, α_2) *admissible* if and only if $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$.[3]
- The individual agent's wealth in the two states of nature (accident or no accident) is represented by a vector (W_1, W_2). If there is no accident, the agent's wealth is the endowment minus the premium paid to the insurer,

$$W_1 = W - \alpha_1. \tag{1}$$

If an accident occurs, he suffers a loss d and receives a reimbursement $\alpha_1 + \alpha_2$,

$$W_2 = W - d + \alpha_2. \tag{2}$$

- Insurers behave as if they are risk neutral. RS argue that insurance company shareholders hold well-diversified portfolios.
- The equilibrium is defined as a set of contracts such that, when customers choose contracts to maximize expected utility: (1) no contract in the equilibrium set makes negative expected profits, and (2) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit.[4]
- All insurance customers have absolute risk aversion,

$$r(x) = -\frac{U''(x)}{U'(x)}, \tag{3}$$

where $r(x)$ may be a constant, a decreasing or an increasing function of wealth x. In this article, when studying the role of risk aversion, $r(x)$ is assumed to be constant in order to control for wealth effects. We postulate that $r(x)$ decreases with wealth while discussing the effect of heterogeneous wealth.
- The expected utility of a risk-type t individual (t = H, L), who purchases contract α (α_1, α_2) is

$$V^t(\alpha_1, \alpha_2) = (1 - p^t)U(W - \alpha_1) + p^t U(W - d + \alpha_2). \tag{4}$$

From (4), the marginal rate of substitution for type t is

$$\mathrm{MRS}^t = -\frac{dW_2}{dW_1} = \frac{(1 - p^t)U'(W_1)}{p^t U'(W_2)}. \tag{5}$$

[3] This restriction is necessary since without it the optimal premium could become negative when transaction costs are extremely high.

[4] The main result of this article, no externality equilibrium, relies on envy-free allocations; any such allocation is an equilibrium whatever the concept (among standard ones). In this respect, most of the article is robust to alternative equilibrium concepts.

It is clear to see that $\frac{1-p^{\mathrm{L}}}{p^{\mathrm{L}}} > \frac{1-p^{\mathrm{H}}}{p^{\mathrm{H}}}$; the low-risk indifference curve is everywhere steeper than the high-risk indifference curve.

TRANSACTION COSTS

In the RS model, there are no transaction costs and insurance firms make zero expected profits. Consequently, for each insurance policy, the expected claim, $p(\alpha_1 + \alpha_2)$, is equal to the premium α_1. The insurer's expected profit can be expressed as

$$\pi = \alpha_1 - p(\alpha_1 + \alpha_2) = 0. \tag{6}$$

In reality, insurance contracts cost more than the expected value of their claims. Allard, Cresta, and Rochet (1997) modify the RS model by introducing a distribution cost, which corresponds to the cost of designing and marketing an insurance contract. The key feature of their model is that the total cost is assumed to be a constant, and therefore the per-customer cost decreases when more customers buy the contract. This economy of scale provides a counterforce to adverse selection, and it leads to their most important conclusion: "Pooling equilibria always exist when the set-up cost is large enough."

In this article, the cost structure has both a constant term and terms that are proportional to their premium or reimbursement:

$$\text{cost} = a + b\alpha_1 + cp(\alpha_1 + \alpha_2). \tag{7}$$

The cost function can be added to the right side of the budget Equation (6). Below, we discuss the constant and proportional components of the general cost function, and analyze their respective impacts on insurance buyers' behavior.

Constant Cost

Insurance companies have many fixed costs, such as rent, technical hardware and maintenance, and utilities. Let us first look at the effect of having a constant cost alone,

$$\text{cost} = a. \tag{8}$$

With a fixed cost, the individual's optimization problem is (hereafter Problem I)[5]

$$\text{Max}: \; E(U) = (1-p)U(W-\alpha_1) + pU(W-d+\alpha_2)$$

[5] In Problem I, the first condition is the budget constraint and the second condition states that people need to see an improvement in their welfare before they pay for any amount of insurance. The final condition guarantees the insurance premium will not become negative. The detailed solution to Problem I is available from the authors on request.

$$\text{Subject to:} \quad \alpha_1(1-p) - \alpha_2 p = a$$
$$(1-p)U(W-\alpha_1) + pU(W-d+\alpha_2) \geq (1-p)U(W) + pU(W-d)$$
$$\alpha_1 \geq 0, \quad \alpha_2 \geq 0.$$

Previous studies have shown that full coverage is optimal when costs are constant. As long as the fixed cost is not too high, people will pay a premium of $pd + a$ and will have wealth $W - pd - a$ regardless of whether an accident occurs. Without any costs, customers' optimal wealth level is $W - pd$, and when there is a fixed cost a, the amount is reduced to $W - pd - a$. We can view the constant load as a reduction in endowment; all insurance buyers start with reduced assets and make the same choice as in a perfect market without transaction costs.

People will not buy any insurance if the fixed cost is too high. An interesting case emerges when the fixed cost is such that the low risks are forced out of the market but the high risks still stay. If this happens, the high risks get their first best contract and the low-risk contract in the second best is the same as in the first best. The high risks do not bring any negative effects to the low risks and the market reaches a special separating equilibrium.

Proportional Cost

Insurance companies also have variable costs, such as commissions, premium taxes, claim adjustment expenses, and litigation expenses, so we can assume that costs also include a variable component. An example of these costs is premium taxes, which vary from 2 to 4 percent of the premium, depending on state regulations. Expenses associated with claim settlements are closely correlated with the total claim amount $\alpha_1 + \alpha_2$.

Consequently, our cost function includes two proportional components:

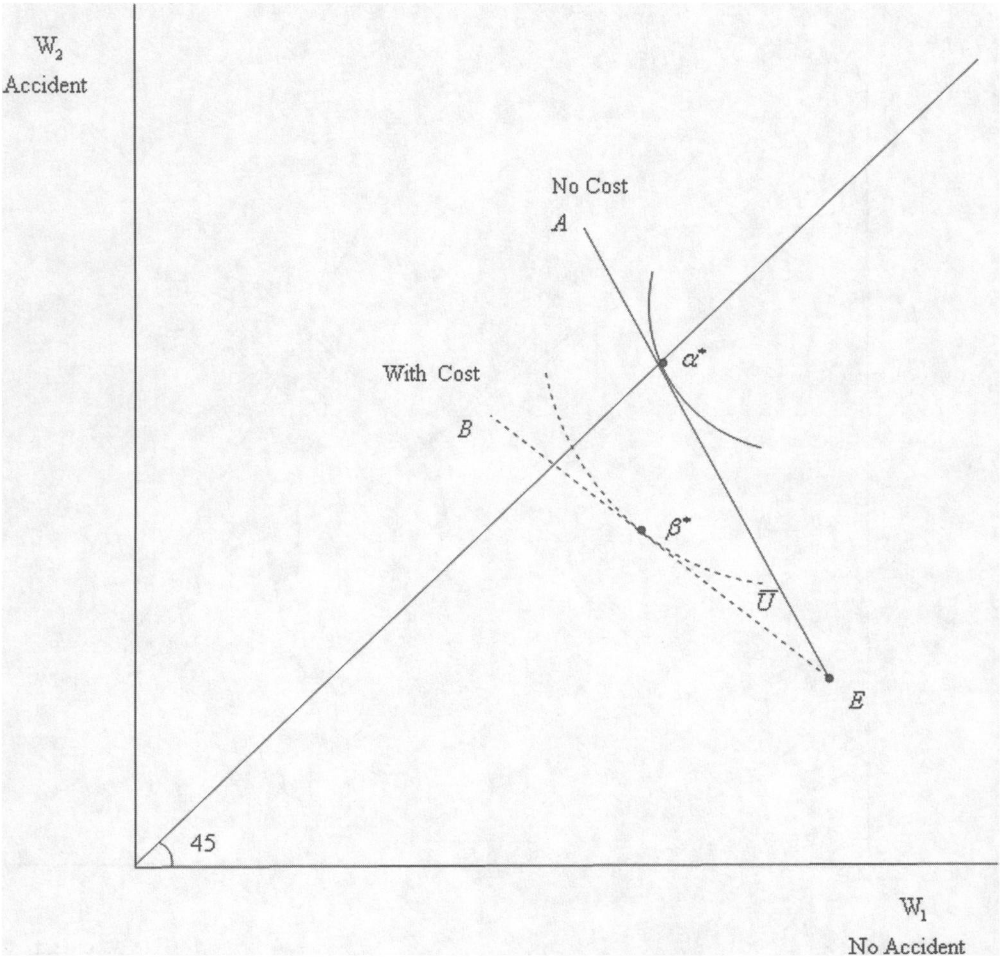
$$\text{Cost} = b\alpha_1 + pc(\alpha_1 + \alpha_2), \tag{9}$$

where b and c are constants. The insurance buyer seeks to maximize his expected utility (Problem II):

$$\text{Max: } E(U) = (1-p)U(W-\alpha_1) + pU(W-d+\alpha_2)$$
$$\text{Subject to: } \alpha_1(1-p) - \alpha_2 p = b\alpha_1 + pc(\alpha_1 + \alpha_2)$$
$$\alpha_1 \geq 0, \quad \alpha_2 \geq 0.$$

Previous studies, such as Smith (1968), conclude that partial insurance is optimal if there are proportional loadings in the premium. In the context of Problem II, partial insurance means that individuals have more wealth in the non-loss state than in the loss state, $W_1^* > W_2^*$. The proof is omitted in the interest of brevity. Figure 1 illustrates why partial insurance becomes optimal. The customer's wealth in the no-accident state is on the x-axis and his wealth in the accident state is on the y-axis. The 45-degree line represents full coverage: the buyer's wealth is the same in both states of nature.

FIGURE 1
Optimal Choice Under Proportional Costs

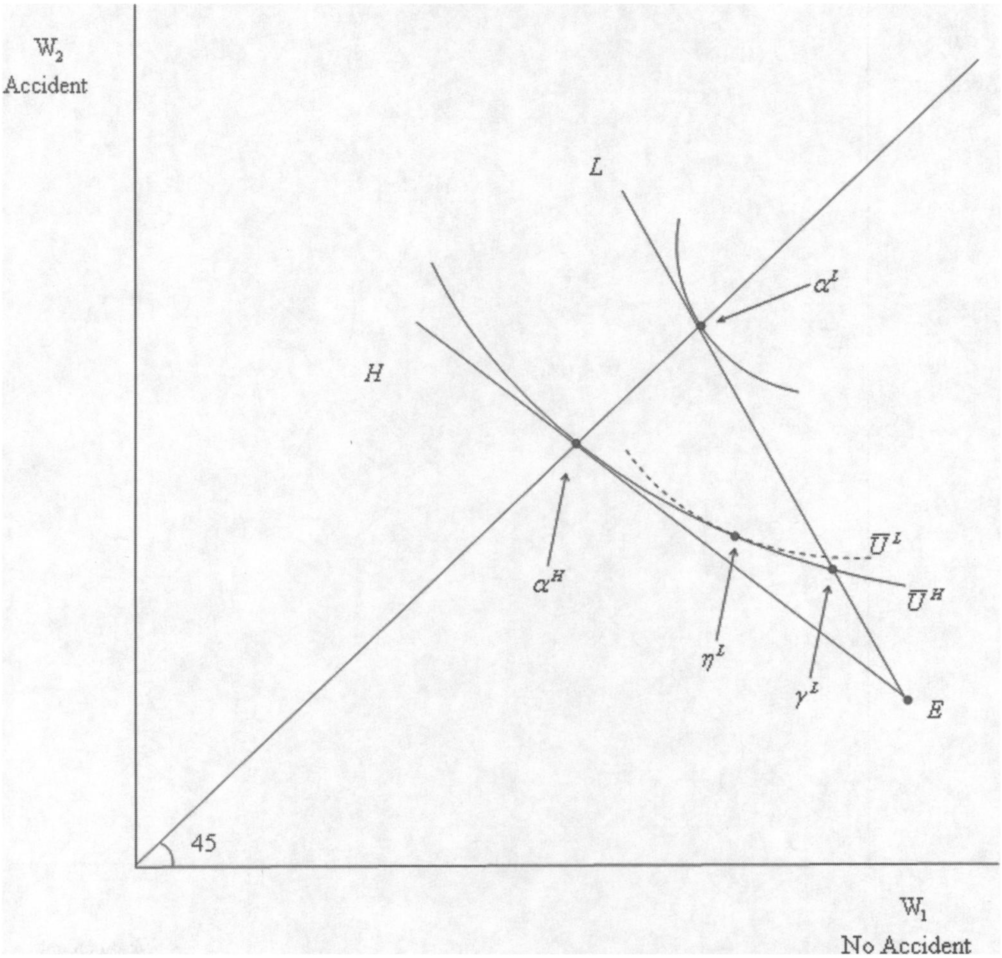


The customer's endowment is *E*. The lines AE and BE are budget, or market odds lines. Their slope is the dollar coverage per premium dollar. Proportional transaction costs reduce the coverage per premium dollar, causing the budget lines to flatten out. High-risk customers face a shallow slope; they get less coverage per premium dollar than do low-risk customers. When there is no cost, the customer buys full coverage, α^*. When there is a proportional cost, the customer's utility curve is tangent to the more shallow budget line at a point β^*, below the full coverage line; the customer buys only partial coverage. Without full coverage, the customer is worse off if an accident occurs.

Externality Under Proportional Costs

Let us first recall the situation in the RS model without transaction costs. In the absence of asymmetric information, both risk types take full insurance α^H and α^L, as shown

FIGURE 2
Separating Equilibrium in the Original RS Framework

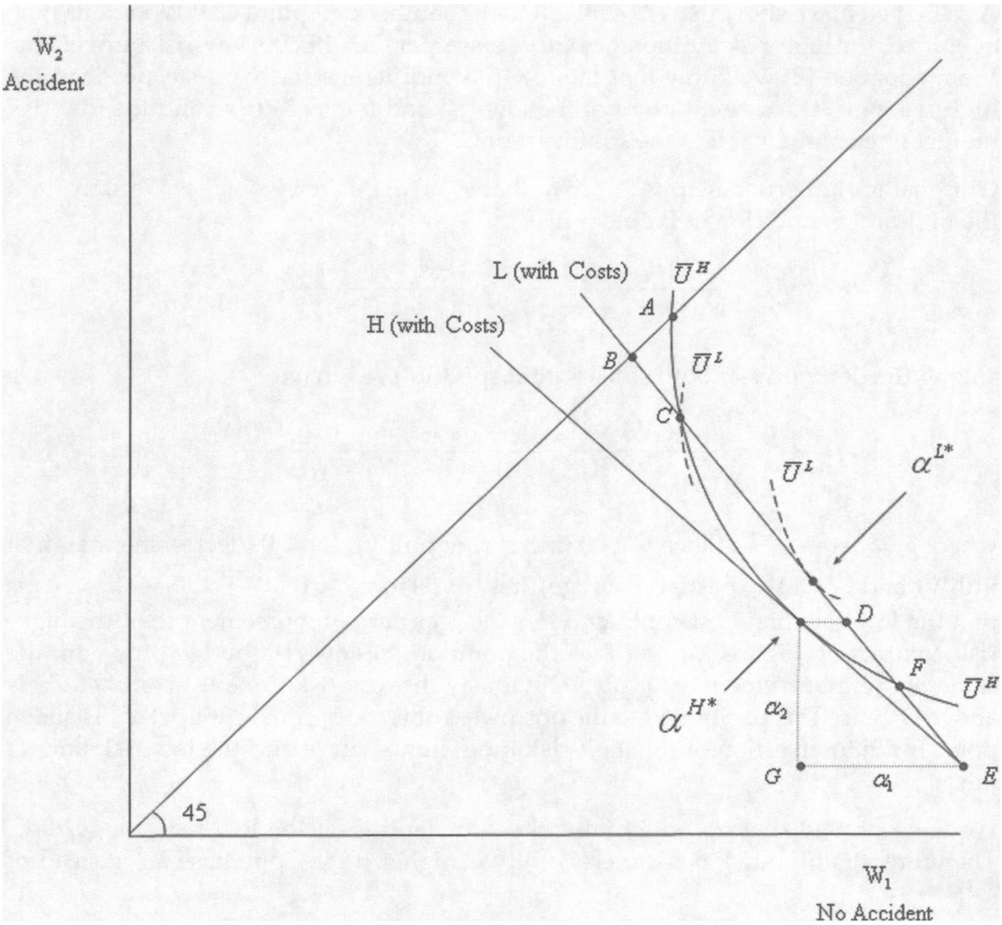


in Figure 2, as their first-best choice. With asymmetric information, high-risk buyers prefer the low-risk contract α^L because it provides more consumption in both states of nature. Since insurers cannot sort out bad customers from good, they offer only a partial coverage contract to the low-risk customers. This contract, γ^L, is at the intersection of the high-risk indifference curve and the low-risk budget line. Thus, the presence of high-risk individuals inflicts a negative externality on low-risk individuals.

Under proportional costs, the optimal contracts for both risk groups are below the 45-degree line in Figure 2. Whether the incorporation of transactions costs can reverse the externality conclusion becomes interesting. Dionne, Gourieroux, and Vanasse (1998) (DGV) conduct the first study on adverse selection with transaction costs. They analyze optimal deductibles under the proportional cost function

$$\text{Cost} = cp(\alpha_1 + \alpha_2), \tag{10}$$

FIGURE 3
Externality in the RS Model With Proportional Transaction Costs



and their conclusion is that the RS externality result persists. To provide additional insight, we use an alternative approach to show the cost effects under the more general cost structure $b\alpha_1 + pc(\alpha_1 + \alpha_2)$.

Proposition 1: *Under the cost function $b\alpha_1 + cp(\alpha_1 + \alpha_2)$, high-risk individuals prefer the low-risk first-best contract to their first-best contract.*

Proof: Our mainly graphical procedure is illustrated in Figure 3. Point E is the insurance buyer's endowment point, and lines LE and HE respectively are the low- and high-risk budget lines with proportional costs. The high-risk indifference curve passes through points A, C, and F, and it is tangent to the high-risk budget line at the point α^{H^*}, the high-risk first-best contract. Our goal is to show that the low-risk first-best contract α^{L^*} must fall between C and F.[6] Therefore, the high-risk customers prefer α^{L^*} to α^{H^*}.

As a first step, we show that α^{L^*} will fall to the southeast of point C. It is obvious that, at point C, the high-risk indifference curve is steeper than LE, the low-risk budget line. From equation (5) we know that the low-risk indifference curve is steeper than the high-risk one. The tangent point of the low-risk indifference curve and the low-risk budget line α^{L^*} must fall to the southeast of C.

Our goal in the second step is to prove that α^{L^*} will be above point F. We start from the first-order condition in Problem II:

$$\frac{U'(W-\alpha_1(p))}{U'(W-d+\alpha_2(p))} = \frac{1-b-p(1+c)}{(1+c)(1-p)}. \tag{11}$$

Taking the derivative of both sides with respect to p, we have

$$\frac{d\alpha_2^*}{dp} = \frac{g[U'(W_2)-U'(W_1)]-(1-p)(\alpha_1+\alpha_2)U''(W_1)}{g^2U''(W_2)+p(1-p)U''(W_1)}, \tag{12}$$

where $g = \frac{1-b-p(1+c)}{1+c}$. Since $U' > 0$ (increasing utility), $U'' < 0$ (decreasing marginal utility), and $W_1 > W_2$ (partial coverage), it is trivial to show that $\frac{d\alpha_2^*}{dp} < 0$, which means that the low-risk first-best contract offers a larger net reimbursement than the high-risk contract. In Figure 3, point D is the point on the low-risk budget line with the same net reimbursement α_2 as α^{H^*}. Obviously the low-risk first-best contract α^{L^*} is above D. Since D is positioned to the northwest of F, the optimal contract α^{L^*} is above point F, the intersection of the high-risk indifference curve and the low-risk budget line.

We have proved that α^{L^*} must fall between C and F on the low-risk budget line. Therefore, the high-risk consumers' utility is higher if they purchase α^{L^*} instead of α^{H^*}. □

Transaction costs are much like taxes imposed on insurance buyers. All else equal, high-risk individuals pay higher premiums (receive less net reimbursement), α_1 (α_2), than do low-risk individuals. Under the cost structure $b\alpha_1 + cp(\alpha_1+\alpha_2)$, high-risk individuals suffer more from the proportional cost, and their coverage is reduced to a greater extent than low-risk individuals' coverage. Therefore, the low-risk contract is more attractive to the high-risk individuals. Moreover, it is trivial to show the RS externality result holds under other conventional transaction cost structures, such as $b\alpha_1$ and $cp(\alpha_1+\alpha_2)$.[7]

[6] It is also possible that point C, the intersection of the high-risk indifference curve and the low-risk budget line, falls above point A, the intersection of the high-risk indifference curve and the 45-degree line. In that case, the graph is different and it is obvious that α^{L^*} would be to the southeast of point C, the intersection of the high-risk indifference curve and the 45-degree line.

[7] We would like to point out that the RS externality result can be reversed under certain cost schemes. For example, if cost $= c\alpha_2$, the low risks will be affected more than the high risks.

RISK AVERSION

Insurance buyers might have different levels of risk aversion. This possibility has been evaluated in an information asymmetry setting by several previous researchers. RS point out that it is possible that low-risk individuals are less risk averse. RS discuss the possibility that high-risk people are more risk averse than low-risk individuals and conclude that a pooling equilibrium cannot exist. De Meza and Webb (2001) find that low-risk, highly risk-averse individuals tend to buy more coverage than high-risk, less risk-averse individuals. In their setting, a pooling equilibrium is possible. Smart (2000) and Villeneuve (2003) add heterogeneous risk aversion to the RS model. In Smart's model, there are four customer groups: high risks with high risk aversion (hh), high risks with low risk aversion (hl), low risks with high risk aversion (lh), and low risks with low risk aversion (ll). His major conclusion is that the indifference curves of the hl and lh types may cross twice and different risk types may be pooled in one contract. Villeneuve assumes that one risk class is more risk averse than the other risk class in the market. He reaches the conclusion that positive profits are sustainable for the low-risk contract and that random insurance contracts may also exist. Our article differs from Smart and Villeneuve in two critical ways: First, we incorporate transaction costs, in addition to heterogeneous risk aversion, into the RS model. Second and most important, we contest the externality conclusion of the RS model.

In this section, we extend the basic RS model with a new set of assumptions:

1. There are proportional transaction costs:

$$\text{Cost} = b\alpha_1 + pc(\alpha_1 + \alpha_2). \tag{13}$$

2. Insurance customers' utility functions depend on risk aversion. The respective utility functions of the high-risk and low-risk customers are $U^{\mathrm{H}}(x)$ and $U^{\mathrm{L}}(x)$, and their respective risk aversion coefficients are $r^{\mathrm{H}}(x)$ and $r^{\mathrm{L}}(x)$. We examine the two possible scenarios: In Scenario A, high-risk individuals are more risk averse than low-risk individuals, $r(x)^{\mathrm{H}} > r(x)^{\mathrm{L}}$, whereas in Scenario B the relation is reversed.
3. It is usually assumed that absolute risk aversion depends on wealth level. We neutralize the wealth effect by assuming that customers have constant absolute risk aversion (CARA). This assumption is not critical; our result could be easily generalized to other cases. According to Pratt (1964), constant risk aversion generates a negative exponential utility function,

$$U^t(x) = -e^{-r^t x}, \quad t = \mathrm{H}, \mathrm{L}. \tag{14}$$

We find numerical examples in which the high risks are no longer interested in the low-risk contract.

Are These Scenario Assumptions Realistic?

Both Scenario A and Scenario B are reasonable in different circumstances. Customers with a history of serious, extended illnesses in their families often have a relatively high likelihood of contracting these illnesses. It is sensible to think that customers with such family histories would also be more risk averse than others in making their health insurance decisions. This gives us a realistic situation of Scenario A. In alternative circumstances, we can find real-world situations for Scenario B. For example, reckless (high-risk) drivers might be less risk averse, and may tend to buy only minimum coverage, since both recklessness and low coverage are consistent with disregard for risk. Likewise, cautious (low-risk) drivers might also be more risk averse and buy more coverage, since both behaviors are consistent with risk aversion.

Indifference Curves

Following RS, our procedure is mainly graphical. The slope of the indifference curve is represented by the marginal rate of substitution,

$$\mathrm{MRS}^t = -\frac{dW_2}{dW_1} = \frac{(1-p^t)U^{t\prime}(W_1)}{p^t U^{t\prime}(W_2)}, \quad t = \mathrm{H}, \mathrm{L}. \tag{15}$$

Under CARA, MRS is $[(1-p^t)/p^t]e^{-r^t(W_1-W_2)}$. Therefore, the ratio of high-risk MRS and low-risk MRS is

$$\frac{\mathrm{MRS}^{\mathrm{H}}}{\mathrm{MRS}^{\mathrm{L}}} = \left(\frac{1-p^{\mathrm{H}}}{p^{\mathrm{H}}} \Big/ \frac{1-p^{\mathrm{L}}}{p^{\mathrm{L}}}\right) e^{-(r^{\mathrm{H}}-r^{\mathrm{L}})\times(W_1-W_2)}. \tag{16}$$

We wish to determine whether this ratio is greater than one; this will tell us whether the high- or low-risk-aversion customers have a greater MRS. Clearly, the first factor on the right side of the equation, $\frac{1-p^{\mathrm{H}}}{p^{\mathrm{H}}}/\frac{1-p^{\mathrm{L}}}{p^{\mathrm{L}}}$, is positive and less than one. Is the second factor, $e^{-(r^{\mathrm{H}}-r^{\mathrm{L}})\times(W_1-W_2)}$, also less than one? Looking at the exponent of this factor, we know that customers should not have more wealth in the loss state than in the no-loss state, $W_2 \leq W_1$, so the last factor in the exponent is positive. Thus, whether the risk aversion factor in the exponent is greater than or less than one depends on the relative levels of risk aversion of the two risk groups.

Scenario A – High-risk individuals are more risk averse than low-risk individuals.

In this case, $r^{\mathrm{H}} > r^{\mathrm{L}}$. So $e^{-(r^H-r^L)\times(W_1-W_2)}$ is less than one and $\mathrm{MRS}^{\mathrm{H}} < \mathrm{MRS}^{\mathrm{L}}$. The high-risk indifference curve is always flatter than the low-risk indifference curve.

Scenario B – High-risk individuals are less risk averse than low-risk individuals.

In this case, $r^{\mathrm{H}} < r^{\mathrm{L}}$ and $e^{-(r^{\mathrm{H}}-r^{\mathrm{L}})\times(W_1-W_2)}$ is greater than one. It is unclear whether the product of the two factors in (16) is greater than or less than one, and therefore, which indifference curve is flatter. If we hold p^{H} and p^{L} fixed, then $e^{-(r^{\mathrm{H}}-r^{\mathrm{L}})\times(W_1-W_2)}$ increases with the difference between r^{H} and r^{L}, and with the difference between W_1

and W_2. When these differences are sufficiently large, $e^{-(r^H - r^L)\times(W_1 - W_2)}$ dominates the first factor and $\frac{MRS^H}{MRS^L} \geq 1$. In this case, the high-risk-indifference curve is steeper than the low-risk-indifference curve. On the other hand, when the differences between r^H and r^L and between W_1 and W_2 are small, the low-risk indifference curve is steeper.

If the indifference curve of one risk type is always flatter, as in Scenario A, the two indifference curves will cross only once. However, if the indifference curves of each risk type are flatter in some circumstances and steeper in others, as in Scenario B, the indifference curves may cross twice.

Market Equilibrium Under Heterogeneous Risk Aversion and No Transaction Costs

Under these assumptions, we find that, in most circumstances, there cannot be a pooling equilibrium, which is consistent with RS. RS note that at any possible pooling point the two indifference curves have different slopes. Any new contract that lies between the two indifference curves will attract the low-risk type away from the pooling point, making the potential pooling equilibrium unstable. In our Scenario A, the slope of the low-risk indifference curve is steeper than the high-risk slope. In Scenario B, it is possible that the low-risk indifference curve is flatter than the high-risk indifference curve. In this case, we can find a destabilizing contract that lies to the west of the point of intersection, above the low-risk indifference curve and below the high-risk indifference curve, as shown in Figure 2 of the original RS paper. This contract will attract the low-risk individuals away from the potential pooling point, making a pooling equilibrium impossible to sustain. In Figure 2, η^L stands for a possible low-risk equilibrium contract if the low-risk indifference curve is flatter than that of the high-risks', which scenario is discussed later in the article.

A special pooling equilibrium can emerge, however, when the two indifference curves are tangent to each other. This case can only be found in Scenario B and it requires that the high-risk indifference curve lies below the low-risk indifference curve. Moreover, it is also necessary that the tangent point falls on the aggregate budget line, which makes zero profit when both high- and low-risk individuals buy one contract. Any contract that lies between the two indifference curves will attract only the high-risk individuals, and it will lead to a negative profit. As noted by Wambach (2000), this type of pooling equilibrium is rare since it calls for special parameter values.

When it comes to the separating equilibrium, the introduction of heterogeneous risk aversion alone does not change the RS argument. The main reason for this is that both risk types take full insurance as their first-best choice. Thus, as in the cases discussed in the "Transaction Costs" section, the presence of high-risk individuals inflicts a negative externality on low-risk individuals.[8]

[8] Villeneuve (2003) suggests that, in Scenario B, the low-risk-indifference curve might be flatter than that of the high-risk group at γ^L, as in Figure 2. In this case, the low-risk group will prefer another point η^L, the tangent point of the two indifference curves. The contract $\eta^L$is profitable to the insurer since it is located below the low-risk budget line.

Separating Equilibrium With Heterogeneous Risk Aversion and Proportional Costs

Overall, the new separating equilibrium under proportional costs and heterogeneous risk aversion differs from the RS equilibrium in two ways. First, both risk groups prefer to have less than full coverage; their optimal contracts are below the 45-degree line. This is due to the presence of proportional costs. Second, we know from Schlesinger (2000) that with proportional costs, whichever group has low risk aversion prefers less insurance; their optional contract is closer to the endowment on the budget line.

The consumer's optimizing decision is as follows. For a type t individual ($t = \mathrm{H, L}$) the optimal contract $\alpha^{t^*}(\alpha_1, \alpha_2)$ is defined as the solution to the following problem (Problem III):

$$\text{Max: } E(U) = (1 - p^t)U^t(W - \alpha_1) + p^t U^t(W - d + \alpha_2)$$

$$\text{Subject to: } \alpha_1(1 - p^t) - \alpha_2 p^t = b\alpha_1 + pc(\alpha_1 + \alpha_2)$$

$$\alpha_1 \geq 0, \quad \text{and} \quad \alpha_2 \geq 0.$$

The expected utility of a type s ($s = \mathrm{H, L}$) individual from a type t's optimal contract is expressed as $V^s\ (\alpha^{t^*})$. For example, a high-risk individual's expected utility from the low-risk optimal contract is

$$V^{\mathrm{H}}[\alpha^{\mathrm{L}^*}] = (1 - p^{\mathrm{H}})U\big[W - \alpha_1^*(p^{\mathrm{L}})\big] + p^{\mathrm{H}}U\big[W - d + \alpha_2^*(p^{\mathrm{L}})\big].$$

Definition 1: *A separating equilibrium in which neither group imposes a negative externality to the other group, which we call an NE (no externality) equilibrium, satisfies the following conditions:*

$$V^{\mathrm{H}}[\alpha^{\mathrm{H}^*}] \geq V^{\mathrm{H}}[\alpha^{\mathrm{L}^*}] \quad \text{and} \tag{17}$$

$$V^{\mathrm{L}}[\alpha^{\mathrm{L}^*}] \geq V^{\mathrm{L}}[\alpha^{\mathrm{H}^*}]. \tag{18}$$

Definition 1 says that an NE equilibrium exists if each risk type prefers the contract designed for their type. The insurance market reaches a separating equilibrium because of self-selection by risk type, and neither risk group imposes a negative externality on the others.

Lemma 1: $V^{\mathrm{L}}[\alpha^{\mathrm{L}^*}] \geq V^{\mathrm{L}}[\alpha^{\mathrm{H}^*}]$ *always holds in the presence of a linear cost function.*

Proof: See the Appendix.

Lemma 1 indicates that if there are proportional costs, low-risk individuals will always prefer the contract designed for their risk group, and thus condition (18) is always true. Therefore, to show that a separating equilibrium exists, it is only necessary to show that high-risk customers prefer their contract, condition (17).

The Existence of an NE Equilibrium.

Proposition 3: *With a proportional cost function,* Cost $= b\alpha_1 + pc(\alpha_1 + \alpha_2)$, *an NE equilibrium exists in Scenario A under condition 1 below. In Scenario B, both conditions 1 and 2 below are needed for an NE equilibrium to exist:*

1. *The difference in risk aversion between the groups is substantial.*
2. *The difference in loss probabilities between the groups is small.*

Proof: We first consider Scenario A, in which high-risk individuals are more risk averse than low-risk individuals.

- Identify the high-risk group's optimal coverage contract, α^{H^*}. See Figure 4.
- The high-risk indifference curve that passes through α^{H^*} must intersect the low-risk budget line at some point; call it C.
- As the low-risk customer's risk aversion declines, he will buy less coverage. In Figure 4, the low-risk optimal contract, α^{L^*1}, moves southeast, toward the endowment point, E.
- We can always find a risk-aversion level for the low-risk-aversion customer that is low enough to make α^{L^*1} fall between C and E. When this happens, high-risk individuals prefer the contract designed for them, α^{H^*}, rather than the contract designed for the low-risk group, α^{L^*1}.
- Therefore, an NE equilibrium exists for Scenario A when the difference between the risk aversion of the two risk groups is large enough.

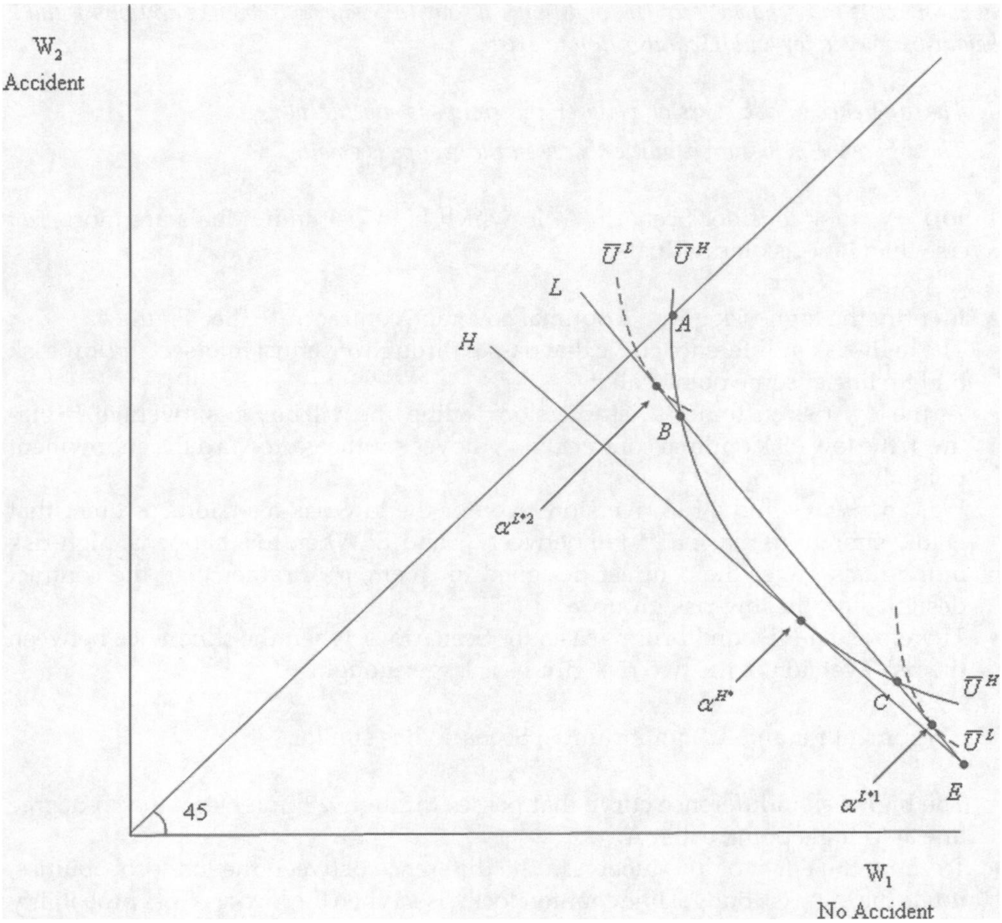
The argument for an NE equilibrium in Scenario B is similar.

- The high-risk indifference curve that passes through α^{H^*} intersects the 45-degree line at a single point; call it A.
- To show the effect of a reduction in the difference between the loss probabilities, rotate the low-risk budget line counterclockwise. When the low-risk loss probability is close enough to the high-risk loss probability, the low-risk budget line intersects the 45-degree line at a point below A and it intersects the high-risk indifference curve at another point, which we call B.
- As the low-risk group's risk aversion increases, low-risk customers buy more insurance coverage and the optimal point, α^{L^*2}, moves toward the 45-degree line. At a certain level of risk aversion, α^{L^*2} falls above B on the low-risk budget line. When this occurs, we have an NE equilibrium. □

In summary, we find that when high-risk customers are more risk averse than low-risk customers, an NE equilibrium exists if the difference in risk aversion between the risk groups is great enough. On the other hand, if high-risk customers are less risk averse, then an NE equilibrium exists if the difference in loss probabilities is small enough and if the difference in risk aversion is great enough.

As we discussed earlier, a pooling equilibrium can emerge in Scenario B under special parameter values. However, a simple argument establishes the fact that a pooling equilibrium cannot exist when there is an NE equilibrium; the low-risk individuals

FIGURE 4
NE Equilibrium With Proportional Costs and Heterogeneous Risk Aversion



prefers the contract α^{L*2}, as in Figure 4, to any other contract since it is the first-best contract under the low-risk budget constraint. Any potential pooling contract is unstable; the low-risk individuals prefer α^{L*2} to the pooling contract and will choose to leave. The contract α^{L*2} has no appeal to the high-risk individuals since, to a high-risk customer, α^{H*} is better than α^{L*2} when an NE equilibrium exists.

In the insurance markets, risk categorization, in which insurers use observable traits that are related to risk for separating customers into risk groups, is often used to mitigate adverse selection. When the loss probabilities are close, as discussed in the NE equilibrium in Scenario B, risk classification is probably of limited practical value; not only are the risk levels too close to distinguish but the risk categorization variables are not particularly sensitive to other variables, such as risk aversion. Risk categorization is most important when consumers do not differ much on dimensions other than risk.[9]

[9] We thank an anonymous referee for suggesting this point.

Will an NE equilibrium exist within a normal range of risk-aversion parameters? Empirical studies of the magnitude of relative risk aversion indicate that the level of risk aversion varies substantially from person to person. In a study of investment behavior, Hansen and Singleton (1993) find that estimates of relative risk aversion range from 0.359 to 58.25. Barsky et al. (1997) find, in an experimental study, that estimates of relative risk aversion range from 0.7 to 15.8. The following example demonstrates that an NE equilibrium may exist within this range of risk-aversion parameters.

Examples of NE Equilibria

Let every customer in the insurance market have initial wealth $W = 1$. By definition, relative risk aversion is $R(W) = -W\frac{U''(W)}{U'(W)}$ and absolute risk aversion is $r(W) = -\frac{U''(W)}{U'(W)}$. Since wealth is set to one, absolute risk aversion has the same range as relative risk aversion. We assume that when an accident occurs, the individual suffers a loss $d = 0.5$, and that both the low- and high-risk customers have negative exponential utility $U(x) = -e^{-rx}$.

First, consider Scenario A: high-risk customers are more risk averse than low-risk customers. Let low-risk individuals have a loss probability $p^{\mathrm{L}} = 0.1$ and high-risk individuals a probability $p^{\mathrm{H}} = 0.15$. Let the absolute risk aversion of the high-risk, high-risk-aversion individuals be $r^{\mathrm{H}} = 2$, and let $r^{\mathrm{L}} = 0.5$ for the low-risk, low-risk-aversion individuals.

If insurance companies provide actuarially fair policies (cost = 0), both low- and high-risk individuals prefer full coverage, and in a transparent market, they are willing to pay premiums of 0.05 and 0.075, respectively. When a loss of 0.5 occurs, customers are fully reimbursed. However, with asymmetric information, the high-risk individuals prefer the low-risk policy. They will gain 0.0077 in expected utility if they buy the low-risk policy. Thus, insurance companies are forced to offer a partial coverage policy to the low-risk customers. The low-risk customers are worse off due to the presence of high-risk customers.

Now, consider the case in which the premium includes a proportional transaction cost $b\alpha_1 + pc(\alpha_1 + \alpha_2)$. Let both b and c be 10 percent. No one will buy full insurance because the premium is not actuarially fair. The high-risk customers pay a premium $\alpha_1^{\mathrm{H}} = 0.07$, compared to 0.075 in the zero-cost case. If a loss $d = 0.5$ occurs, they will receive a reimbursement of 0.38. Low-risk customers will scale back their premium payment α_1^{L} from 0.05 to 0.0059, and in case of a loss, they will recover only 0.049, about 10 percent of the loss amount. Because the low-risk best coverage is much lower than the high-risk best coverage, high-risk individuals receive greater expected utility[10] from their own policy $V^{\mathrm{H}}[\alpha^{\mathrm{H}^*}] = -0.162$ compared to the low-risk policy cut, $V^{\mathrm{H}}[\alpha^{\mathrm{L}^*}] = -0.167$. Low-risk customers also prefer their own policy $V^{\mathrm{L}}[\alpha^{\mathrm{L}^*}] = -0.624$ to the high-risk policy ($V^{\mathrm{L}}[\alpha^{\mathrm{H}^*}] = -0.631$). Thus, self-selection leads to an NE equilibrium.

[10] The expected utility level is negative because $U(x) = -e^{-rx}$. If the reader prefers positive expected utility, it is always permissible to add a fixed positive number. The utility function $U(x)$ is equivalent to the utility function $\tilde{U}(x) = AU(x) + B$, with $A > 0$.

Similarly, under Scenario B, in which low-risk customers are more risk averse, there may also be an NE equilibrium. Let high-risk customers have an absolute risk-aversion coefficient $r^{\mathrm{H}} = 0.5$, and let low-risk customers have a coefficient $r^{\mathrm{L}} = 2$. Assume that high-risk customers have a loss probability $p^{\mathrm{H}} = 0.105$, and that low-risk customers have a loss probability that is close to the low-risk probability, $p^{\mathrm{L}} = 0.1$. In this case, high-risk customers are willing to spend 0.0059 on a premium, whereas low-risk customers will pay a premium of 0.047. In case of a loss, $d = 0.5$, high-risk customers will only receive a payment of 0.046, whereas low-risk customers will recover 0.39. Again, low-risk customers prefer their own policy, $V^{\mathrm{L}}[\alpha^{\mathrm{L}^*}] - V^{\mathrm{L}}[\alpha^{\mathrm{H}^*}] = 0.0047$, and high-risk customers prefer a high-risk policy, $V^{\mathrm{H}}[\alpha^{\mathrm{H}^*}] - V^{\mathrm{H}}[\alpha^{\mathrm{L}^*}] = 0.0002$, and an NE equilibrium exists.

WEALTH

RS assume that all agents have the same endowment, but it is possible that one risk class is wealthier than the other. For example, when use of age is prohibited in health insurance underwriting, young individuals are in general both lower risk and less affluent than middle-aged individuals. Alternatively, disabled individuals may be, on average, both less wealthy and less healthy than the general population. In states such as New Jersey, New York, and Vermont, where use of disability status is prohibited, this gives a perfect example of negative association between risk and wealth level.

When both proportional costs and heterogeneous wealth are added to the RS model, an NE equilibrium may occur. Insurance buyers' absolute risk aversion should vary with wealth and we assume decreasing absolute risk aversion (DARA).[11] In this situation, wealth influences agents' behavior through risk aversion; under DARA, greater wealth is equivalent to lower risk aversion, and *vice versa*. The analyses under heterogeneous wealth are identical to those under heterogeneous risk aversion, and are not repeated in the interest of brevity.

It might be argued that since wealth is probably observable, risk classification could be utilized to control for the divergence in endowment. However, there are two reasons that risk classification may not be relevant in this case. First, we find that a self-selecting NE equilibrium often exists, and risk classification is irrelevant when this occurs. Second, classification is becoming problematic in the United States. Various state laws and regulations prohibit insurers from using certain characteristics, such as a person's sex, age, and physical or mental impairment, to discriminate among individuals applying for insurance.

CONCLUSION

In this article, we investigate several extensions of the RS model of adverse selection in insurance markets. RS assume that markets are without transaction costs and that agents are homogenous except in loss probability. They conclude that at the least, negative externalities harm consumers in one risk category, and at the worst, the market may not be able to reach an equilibrium. We analyze the same problem under

[11] DARA is a commonly accepted assumption in the literature. An alternative assumption is increasing absolute risk aversion (IARA). Individual's behavior would be a mirror image of what we see under DARA, with the roles of the two risk types reversed. Since our result relies on the difference between the two types, the NE equilibrium may also exist under IARA.

the joint assumption that transaction costs are present and that the risk types differ on another dimension (risk aversion or wealth) in addition to risk level. We summarize our study as follows:

1. *Transaction Costs*. Contrary to the assumptions of RS, transaction costs do exist. When proportional transaction costs are present, individuals choose partial coverage.
2. *Adverse Selection*. Inclusion of transaction costs leads to fundamental changes in the dynamic of the RS model. Analysis of informational problems more closely resembles actual market dynamics when transaction costs are considered.
3. *Heterogeneity*. First, insurance buyers may differ in risk aversion depending on their risk types. Practical examples are found in medical insurance and automobile insurance. Moreover, one risk type may also be more wealthy than another risk type. Finally, risk types might have heterogeneous loss severity. This case has been studied by Doherty and Jung (1993), and their conclusion is consistent with ours.
4. *NE Equilibrium*. The interaction of transaction costs and consumer heterogeneity may lead consumers to self-selection by risk type, and in this separating equilibrium there may be no negative externalities.
5. *Risk Categorization*. Our study suggests that risk classification is likely to be most useful when customers do not differ significantly on relevant dimensions other than risk. When they do differ in other ways, then an NE equilibrium may occur, making risk assessment irrelevant.

It has been long noted that RS negative externality and market equilibrium results are not consistent with many observable phenomena and empirical results. Many markets, such as labor and financial markets, function well in the presence of information asymmetry, in contrast to the pessimistic RS prediction. Chiappori and Salanié (2000) find that customers who have higher accident probabilities show no evidence of choosing contracts with more comprehensive coverage. Our results provide some insight into the forces within markets that may partially explain these phenomena. The role of interactions between market characteristics and consumer heterogeneity on multiple dimensions may provide a fruitful avenue for future empirical work in markets characterized by information asymmetry.

APPENDIX

Proof of Lemma 1: We are comparing the value of function $V^{L}[\alpha]$ at two points, $\alpha^{L^*}[\alpha_1(p^L), \alpha_2(p^L)]$ and $\alpha^{H^*}[\alpha_1(p^H), \alpha_2(p^H)]$.

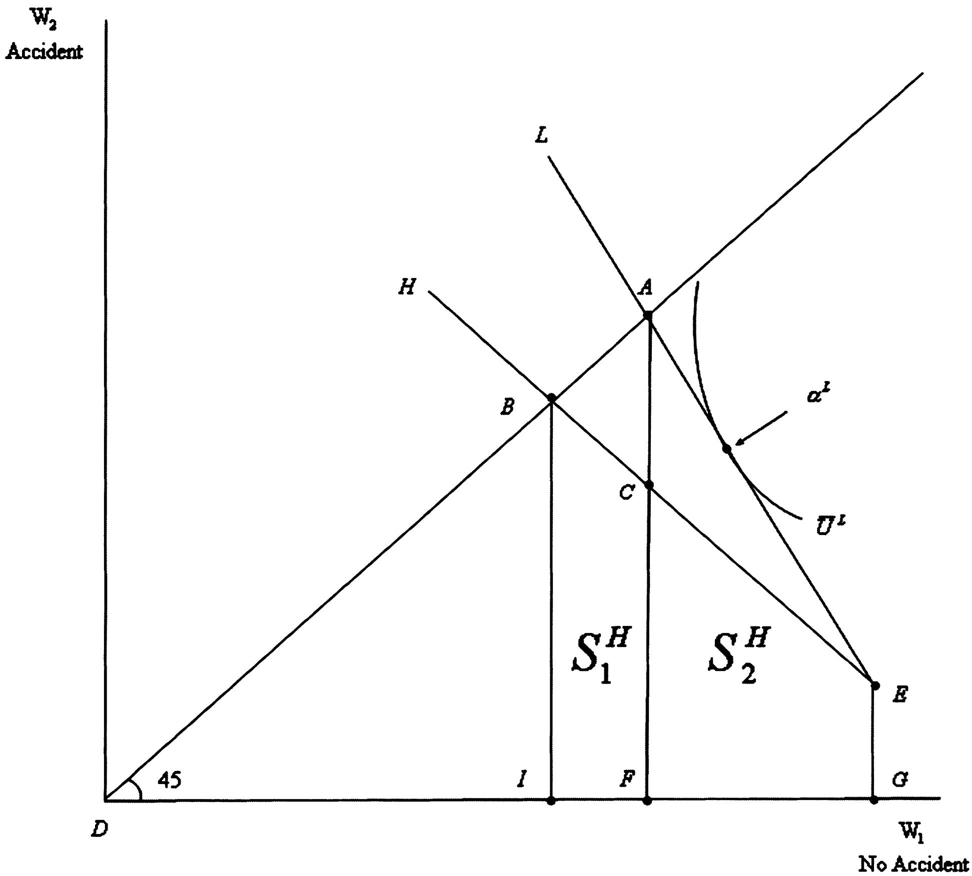
The low-risk individual's optimal solution, α^{L^*}, is the solution to the following problem:

$$\text{Max: } E(U) = (1 - p^L)U(W - \alpha_1) + p^L U(W - d + \alpha_2)$$

$$\text{Subject to: } \alpha_1(1 - p^L) - \alpha_2 p^L = b\alpha_1^L + c \times p^L(\alpha_1^L + \alpha_2^L)$$

$$\alpha_1^L \geq 0, \quad \alpha_2^L \geq 0.$$

FIGURE A1
Low-risk Individuals Do Not Prefer the High-Risk Optimal Contract



We can rewrite the first constraint as

$$\alpha_2^{\mathrm{L}} = \frac{1 - b - p^{\mathrm{L}}(1 + c)}{p^{\mathrm{L}}(1 + c)} \times \alpha_1^{\mathrm{L}}.$$

Since the value of $E(U)$ increases with α_2, we can reformulate Problem I using the following restatement of the first constraint:

$$\text{Max: } E(U) = (1 - p^{\mathrm{L}})U(W - \alpha_1) + p^{\mathrm{L}}U(W - d + \alpha_2)$$

$$\text{Subject to: } \alpha_2^{\mathrm{L}} \leq \frac{1 - b - p^{\mathrm{L}}(1 + c)}{p^{\mathrm{L}}(1 + c)} \times \alpha_1^{\mathrm{L}}$$

$$\alpha_1^{\mathrm{L}} \geq 0, \quad \alpha_2^{\mathrm{L}} \geq 0.$$

Let S^{L} be the set of feasible solutions to this problem. In Figure A1, this set is the area AEGF.

The comparable problem for the high-risk customer is

$$S^{H} \equiv \left\{ (\alpha_1, \alpha_2) \,\middle|\, \alpha_2 \leq \frac{1 - b - p^{H}(1+c)}{p^{H}(1+c)} \times \alpha_1, \alpha_1 \geq 0, \alpha_2 \geq 0 \right\}.$$

This set is the area BEGI in the figure. By definition, $\alpha^{H^*} \in S^{H}$.

It is clear to see that $S_1^{H} \subset S^{L}$.

$V^{L}[\alpha^{L^*}]$ is the point of maximum utility for the function $V^{L}[\alpha]$ over the set S^{L}, and is therefore preferred to all contracts in S^{L}, including point A. Using the notation $X \succ Y$ to indicate that the low-risk customer prefers contract X to contract Y, this result is: $\alpha^{L^*} \succ$ any point in S^{L}, including specifically: $\alpha^{L^*} \succ A$.

Let us divide S^{H} into two subsets: S_1^{H}, which is CEGF, and S_2^{H}, which is BCFI.

Therefore, $S^{H} \equiv S_1^{H} \cup S_2^{H}$.

It is easy to see that low-risk customers prefer α^{L^*} to any contract in set S_1^{H}: $\alpha^{L^*} \succ S_1^{H}$.

Compared to contract A, any contract within S_2^{H} yields less wealth in both states of nature. Therefore, low-risk customers prefer A to any contract in the set S_2^{H}: $A \succ S_2^{H}$.

Therefore,

$\alpha^{L^*} \succ A \succ$ any point in S^{H}, including α^{H*}, and

$$V^{L}[\alpha^{L*}] \geq V^{L}[\alpha^{H*}]. \qquad \square$$

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