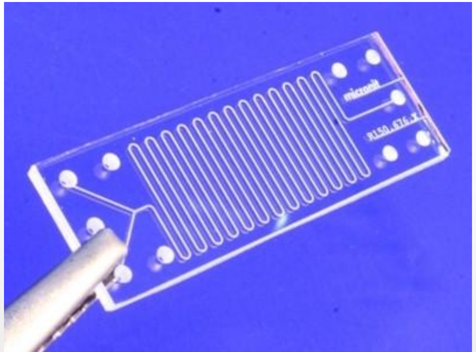
CAV'14 Short Paper Submission

Synthesis of Microfluidics Chips using SMT Solvers

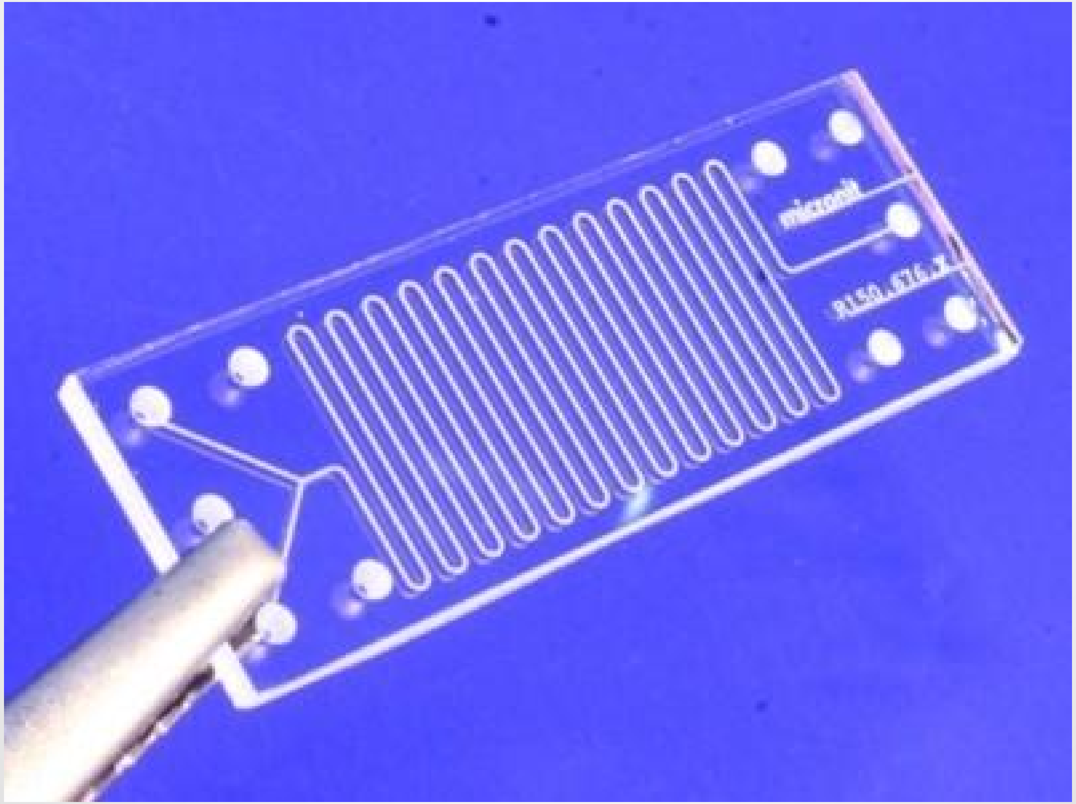
Asif Khan, Derek Rayside, Vijay Ganesh



Example Microfluidic Chip

Channels
with fluids

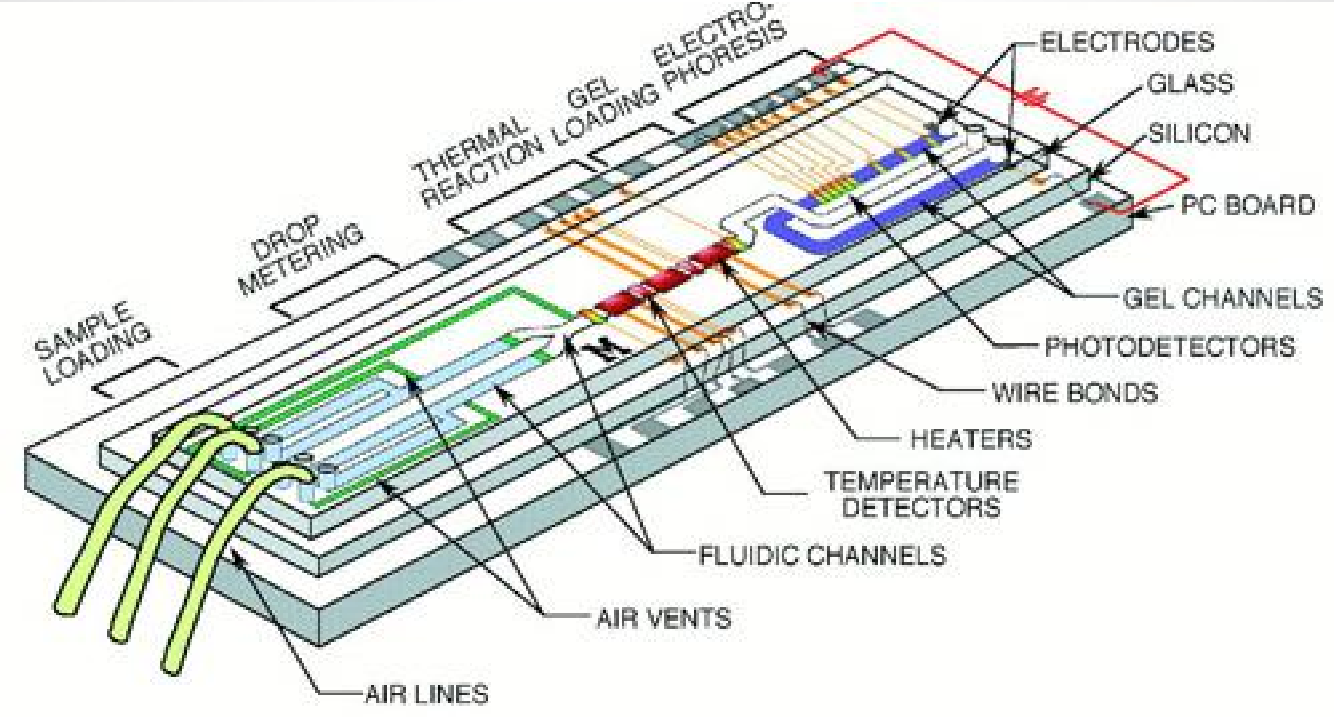
Instead of wires
with electrons.



Test fluid or
Create emulsion

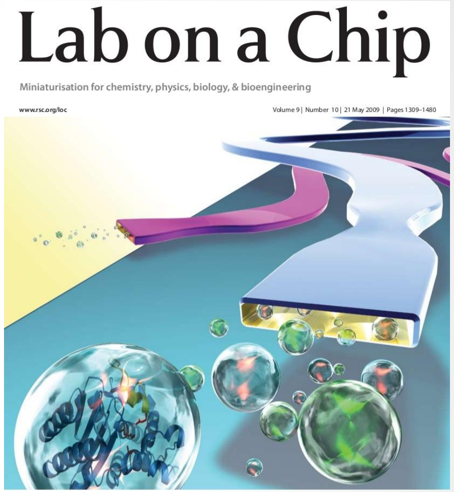
Instead of
computation.

Example Microfluidic Chip Schematic



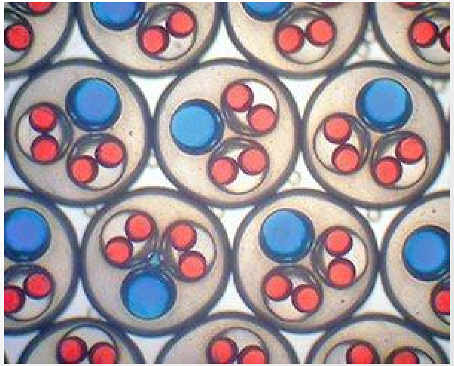
Applications of Microfluidics

- **Lab-on-a-Chip**
  - integrates several lab functions
  - manipulate minute fluid samples
  - performs chemical analysis
  - Pathogen and drug testing
  - Low cost, scalable, recyclable.



Fast!
Cheap!

- **Drops inside drops**
  - New emulsions
  - Targeted drug delivery



Uniform!
Unique!

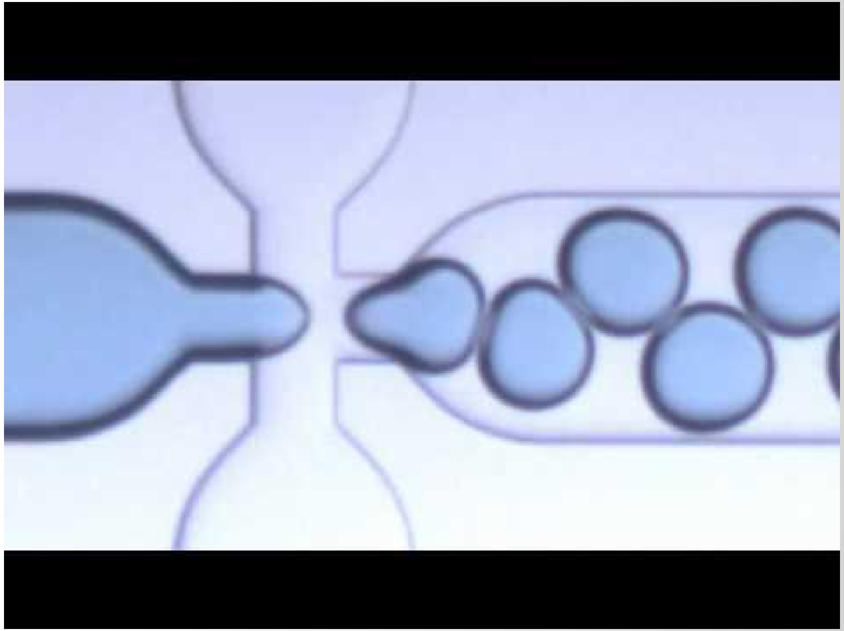
Larger than a speck of dust

Nano < ***Micro*** < Human

Microfluidics uses the same equations as at human scale.
But the impact of the constant factors is different.

Constant Factors

- Higher surface tension
- Fast thermal relaxation
- Diffusion
- Electrowetting
- Initial flow shockwave



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Current Opinion in Biotechnology

Recent developments in microfluidic large scale integration

Ismail Emre Araci[1] and Philip Brisk[2]

Current Opinion in Biotechnology 2014, **25**:60–68

This review comes from a themed issue on **Analytical biotechnology**

Edited by **Frank L Jaksch** and **Savaş Tay**

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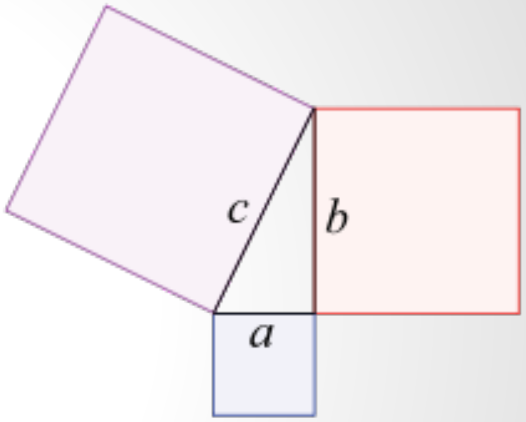
Pressure and Flow Constraints

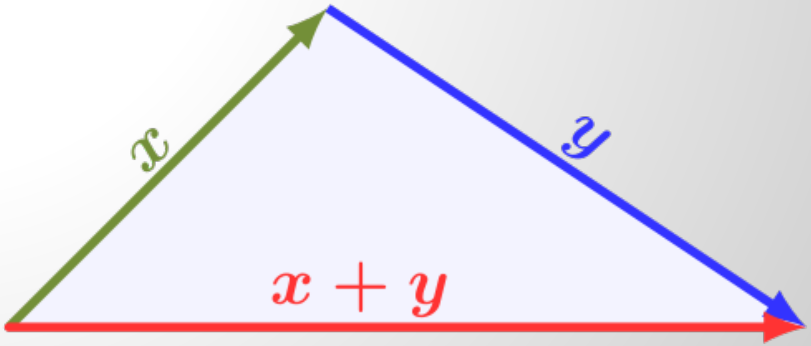
$$P_i = P_j + (-1)^{\phi} Q_{ij} R_{ij}$$

- Pi: pressure at node i
- Qij: rate of flow from node i to node j
- Rij: hydrodynamic resistance in channel
    - defined in terms of channel shape and size

Placement Constraints

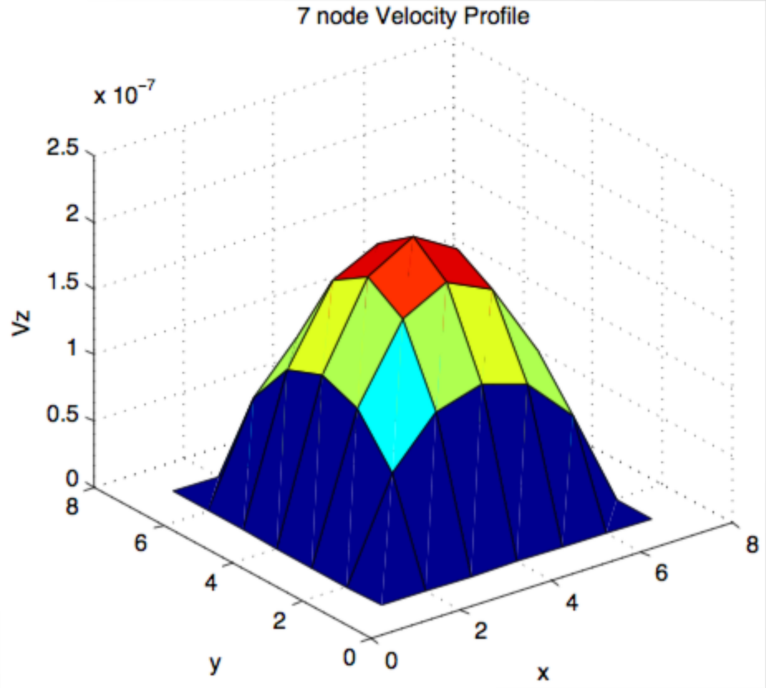
- Desired topology
- External connections
- Manufacturing limits
- Physical limits
- Chip size
- etc.





Velocity Profile of Fluid in Channel

- No-slip boundary (velocity 0 at edge)
- Fastest in centre
- Navier-Stokes eqn



Navier-Stokes Eqn for Velocity Profile

PDE:

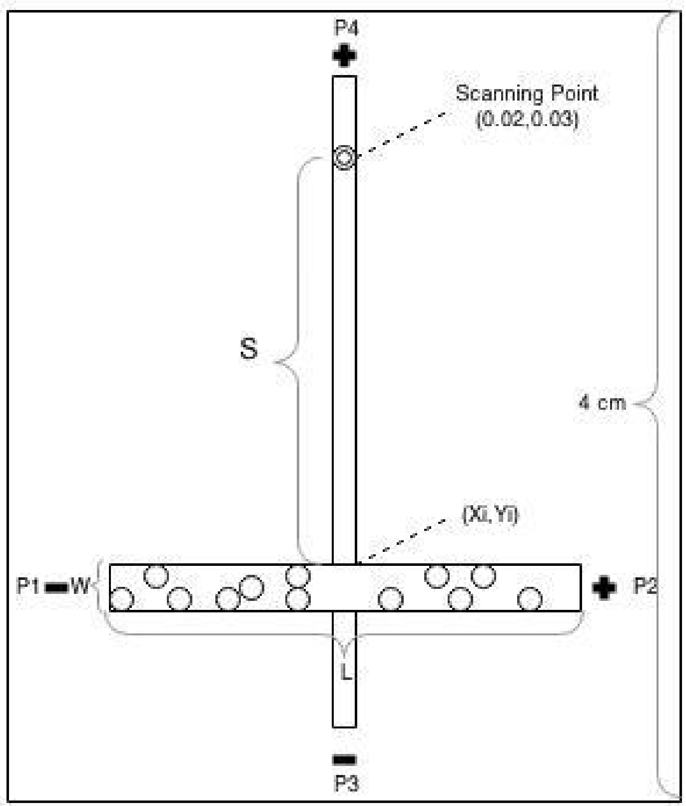
$$\xcancel{0 = 1 + \left(\frac{h}{w}\right)^2 \frac{\partial^2 \widetilde{V}}{\partial \widetilde{x}^2} + \frac{\partial^2 \widetilde{V}}{\partial \widetilde{y}^2}}$$

Finite Element Method:

$$0 = 1 + \frac{\widetilde{V}^j_{i+1} - 2\widetilde{V}^j_i + \widetilde{V}^j_{i-1}}{\Delta \widetilde{x}^2} \left(\frac{h}{w}\right)^2 + \frac{\widetilde{V}^{j+1}_i - 2\widetilde{V}^j_i + \widetilde{V}^{j-1}_i}{\Delta \widetilde{y}^2}$$

Electrophoresis circuit

Detects substance of interest in fluid medium



dReal scales better than Z3 here

Table 1 Timing results for Z3 and dReal solving the example problem.

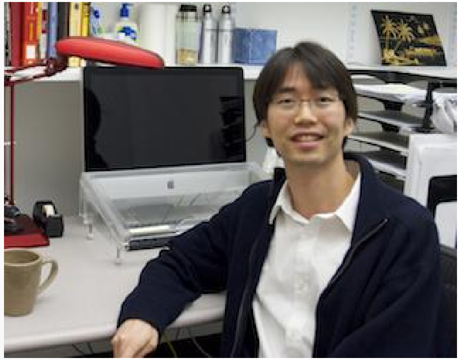
Mesh Size	Z3	dReal	($\delta = 0.002504$)
4^2	4.432	0.264	Times in seconds.
5^2	4.443	0.246	
6^2	27.908	0.438	dReal solutions verified manually
7^2	908.278	0.737	and manually compared with Z3 so-
8^2	· ·	1.215	lutions for validity.
16^2	· ·	22.610	
32^2	· ·	596.963	

dReal: SMT with reals, ODEs, trig

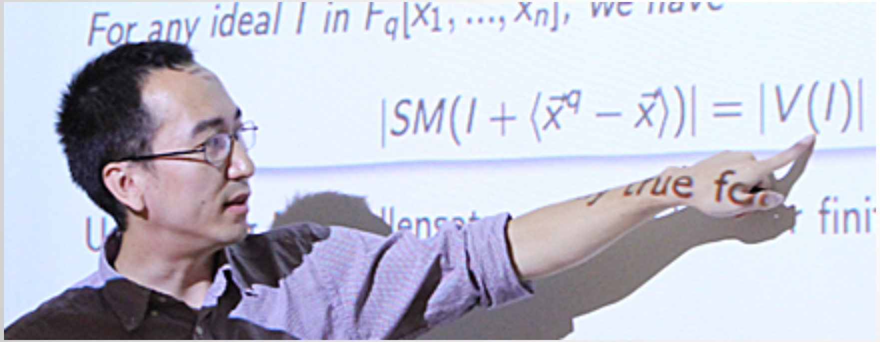
$\cancel{f(x) = 0}$

$f(x) < \delta$

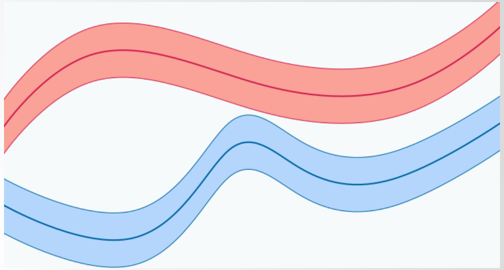
Soonho Kong



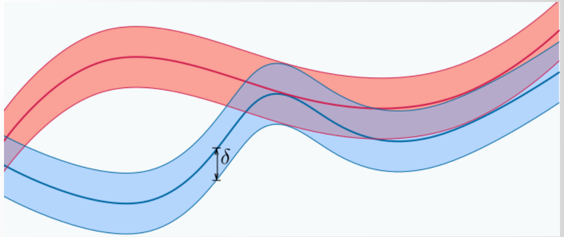
Sean (Sicun) Gao



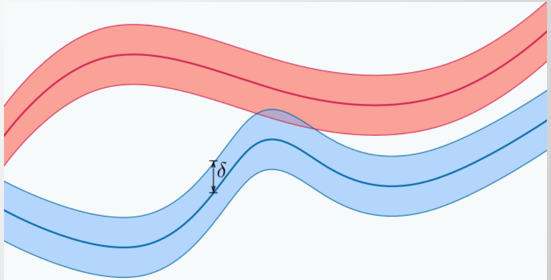
UNSAT



SAT

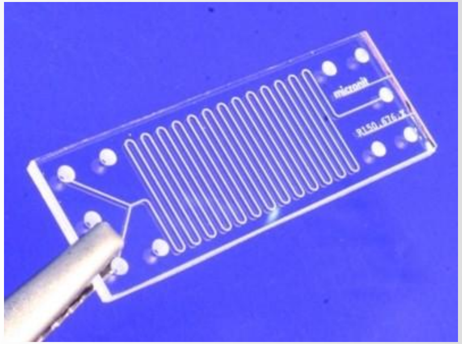


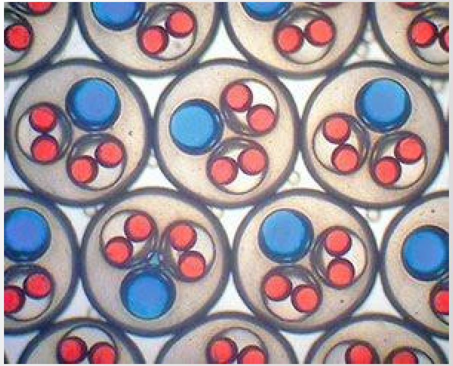
Almost!



Microfluidics needs us!

- Languages!
- Generators!
- Solvers!





Microfluidics vs Digital Circuits

- ***Channels matter:*** shape, diameter, surface area, length, intersection angles, intersection cross-sections, etc.
- ***Mixed circuits:*** e.g., a microfluidic circuit that has an electronic circuit layered on top of it as a controller.
- "***Through***" (e.g., current/flow) and "***across***" (e.g., voltage/pressure) variables.